# Issues on beam dynamics in SuperKEKB: Hard-edge dipole fringe and FMA

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With contributions from

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**Cornell:** D. Sagan

**SLAC:** Y. Cai

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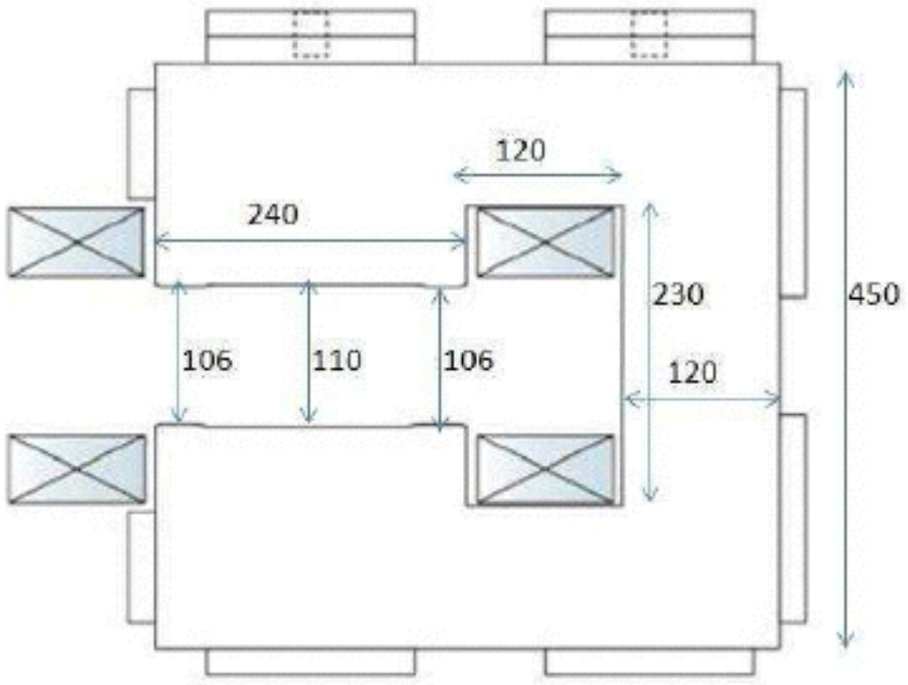
# Outline

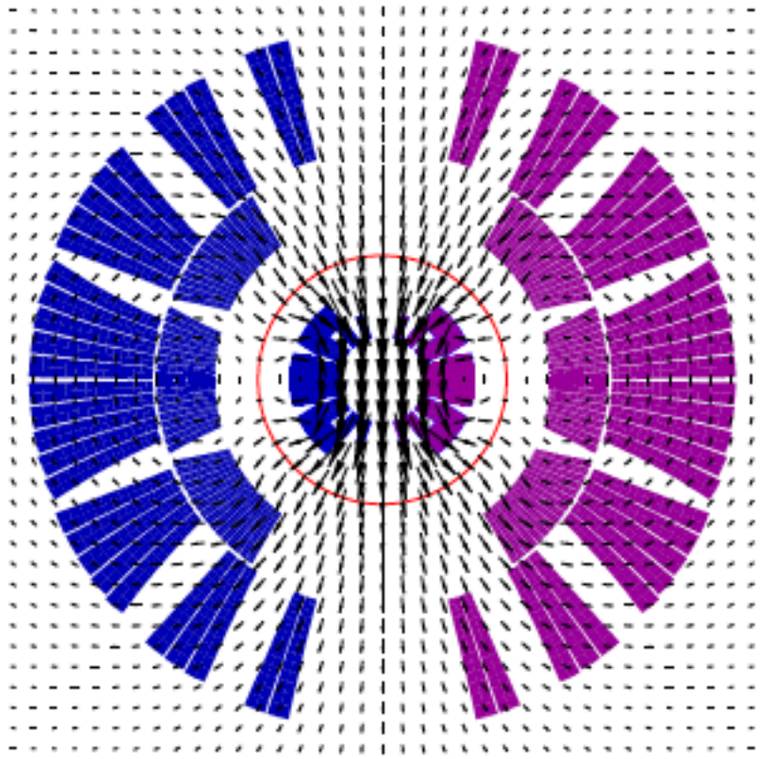
- ➤ **Hard-edge model for dipole fringe field**
  - **Resolve the discrepancy of two models: Cai-Nosochkov's and Forest's**
- ➤ **Benchmark of SAD and Bmad**
  - **FMA results**

# 1. Dipole fringe

- ➤ Question raised from debugging SAD code
- ➤ Two models found for hard-edge fringe
  - E. Forest: “Parallel-plate” shape (popular theory)
  - Y. Cai: Round shape (apply for SC magnets?)




Realistic case
(From SuperKEKB TDR)

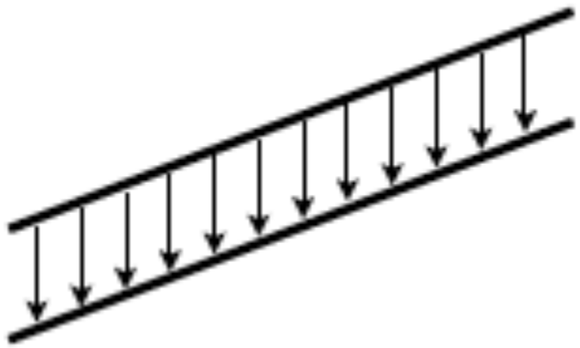


SC magnet
(From S. Russenschuck’s textbook, 2010)

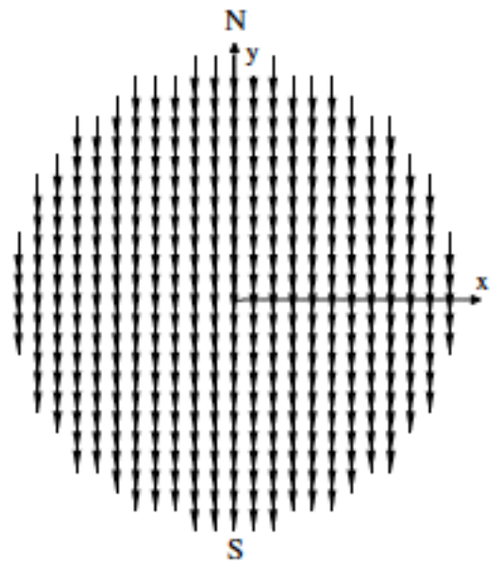
# 1. Dipole fringe

## ➤ Maxwellian solution for hard-edge dipole field

- G. Lee-Whiting et al. => E. Forest et al.
- S. Caspi et al. => M. Bassetti et al. => Y. Cai et al.



The model for wide magnet.
The filed is confined at region of
$-b<y<b$ and $-\infty<x<\infty$




The model for harmonics expansion.
The field is confined inside a circle with $r<r_0$
(From S. Russenschuck's textbook, 2010)

# 1. Dipole fringe

## ➤ Maxwellian solution for hard-edge dipole field

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$$A_s = -xB(s) = -xB_0\theta(s).$$

$$\vec{A} = (A_x, 0, A_s)$$

$$\nabla \times \nabla \times \vec{A} = 0$$

$$A_x = B_0 \sum_{n=1}^{\infty} \frac{(-1)^n \theta^{(2n-1)}(s)}{(2n)!} y^{2n}$$

$$A_y = 0$$

$$A_x = \frac{1}{2}(x^2 - y^2) \sum_{p=0}^{\infty} \frac{1}{2+p} G_{1,2p+1}(s)(x^2+y^2)^p,$$

$$A_y = xy \sum_{p=0}^{\infty} \frac{1}{2+p} G_{1,2p+1}(s)(x^2+y^2)^p,$$

$$A_s = -x \sum_{p=0}^{\infty} G_{1,2p}(s)(x^2+y^2)^p.$$

$$G_{n,2p}(s) = (-1)^p \frac{n!}{4^p(n+p)!p!} \frac{d^{2p} G_{n,0}(s)}{ds^{2p}},$$

$$G_{n,2p+1}(s) = \frac{dG_{n,2p}(s)}{ds},$$

$$A_y \neq 0$$

# 1. Dipole fringe

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**Field distribution with hard-edge:**

$$B_x = 0,$$

$$B_y(y,s) = B_0 \sum_{n=1}^{\infty} \frac{(-1)^n y^{2n} \theta^{(2n)}(s)}{(2n)!},$$

$$B_s(y,s) = -2B_0 \sum_{n=1}^{\infty} \frac{(-1)^n n y^{2n-1} \theta^{(2n-1)}(s)}{(2n)!}.$$

**Field distribution with hard-edge:**

$$B_x(x,y,s) = -\frac{1}{2} B_0 xy \sum_{p=0}^{\infty} \frac{(-1)^p (x^2+p^2)^p \theta^{(2p+2)}(s)}{4^p p!(p+2)!}, \tag{20}$$

$$B_y(x,y,s) = B_0\theta(s) + B_0 \sum_{p=0}^{\infty} \frac{(-1)^{p+1} (x^2+y^2)^p \theta^{(2p+2)}(s)}{4^{p+1}(p+1)!(p+2)!} \left[x^2 + (2p+3)y^2\right], \tag{21}$$

$$B_s(x,y,s) = B_0 y \sum_{n=1}^{\infty} \frac{(-1)^p (x^2+y^2)^p \theta^{(2p+1)}(s)}{4^p p!(p+1)!}. \tag{22}$$

# 1. Dipole fringe: conclusion

➤ Cai-Nosochkov's model applies for round shape (SC magnet is close to this case)

➤ Forest's model applies for wide shape (SuperKEKB dipole is close to this case?)

➤ The realistic magnets are between the two models

# 2. Benchmark of SAD and Bmad

## ➤ Compare FMA using Bmad and SAD

- Same FMA subroutine by D. Zhou
- Examples: sler_1684 and sher_5755
- FMA results: tracking on-momentum particles by varying initial transverse coordinates, plot DA in X-Y space and footprint in tune space.

## ➤ Future work with Bmad and SAD

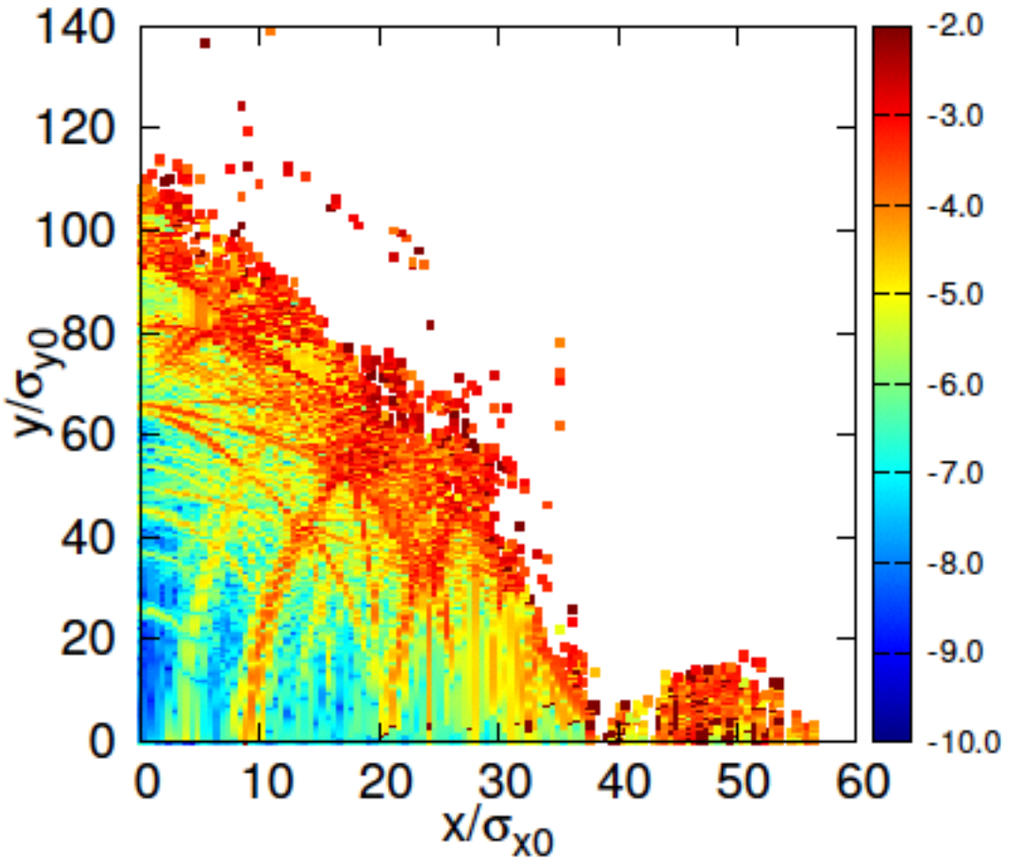
- Try Bmad's FMA subroutine
- Try E. Forest's PTC on FMA

# 2. Comparison of FMA: sler_1684

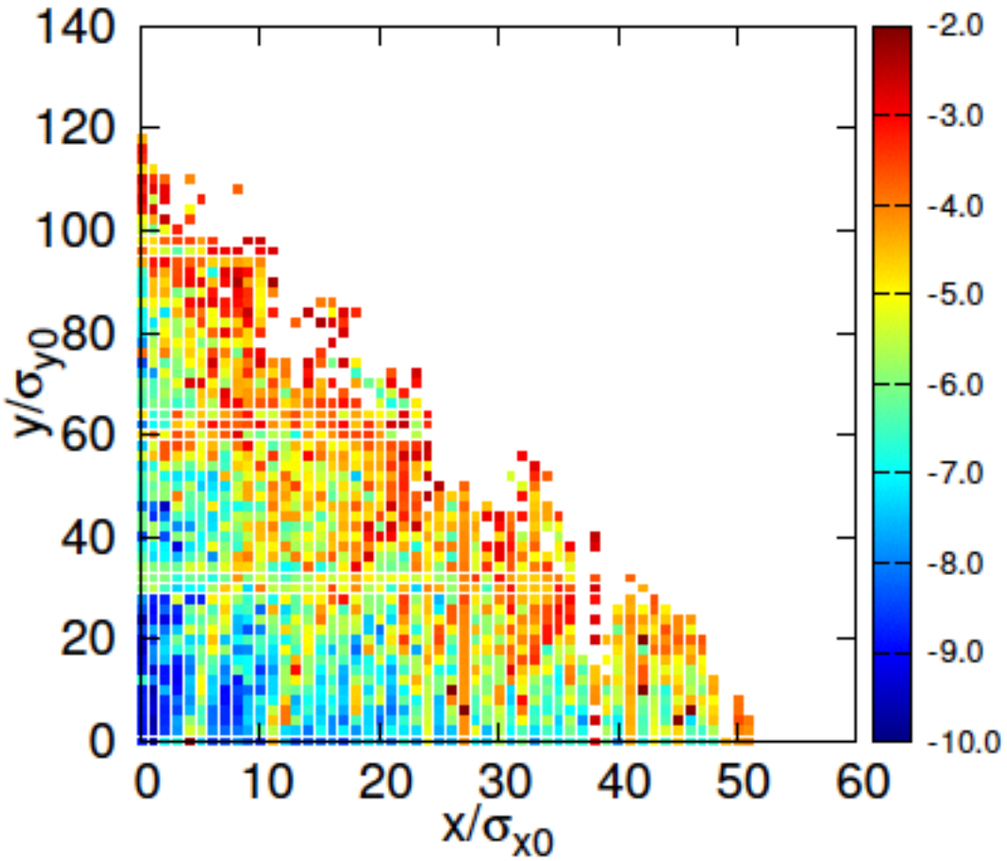
## ➤ X-Y space

- Bmad and SAD give similar DA in size
- Discrepancy exists in details: indicate different (high-order) nonlinear maps used by Bmad and SAD?

Bmad



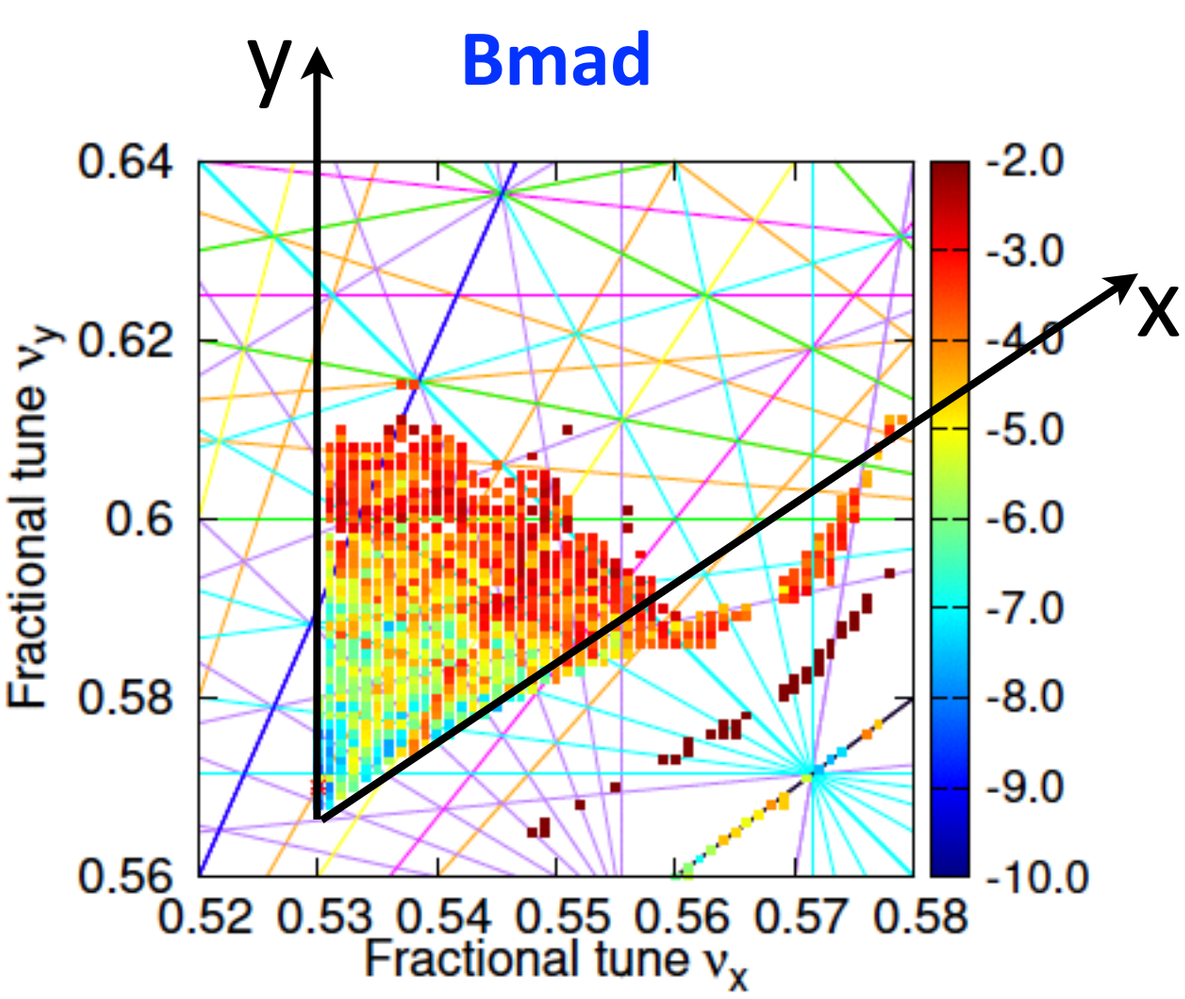

SAD

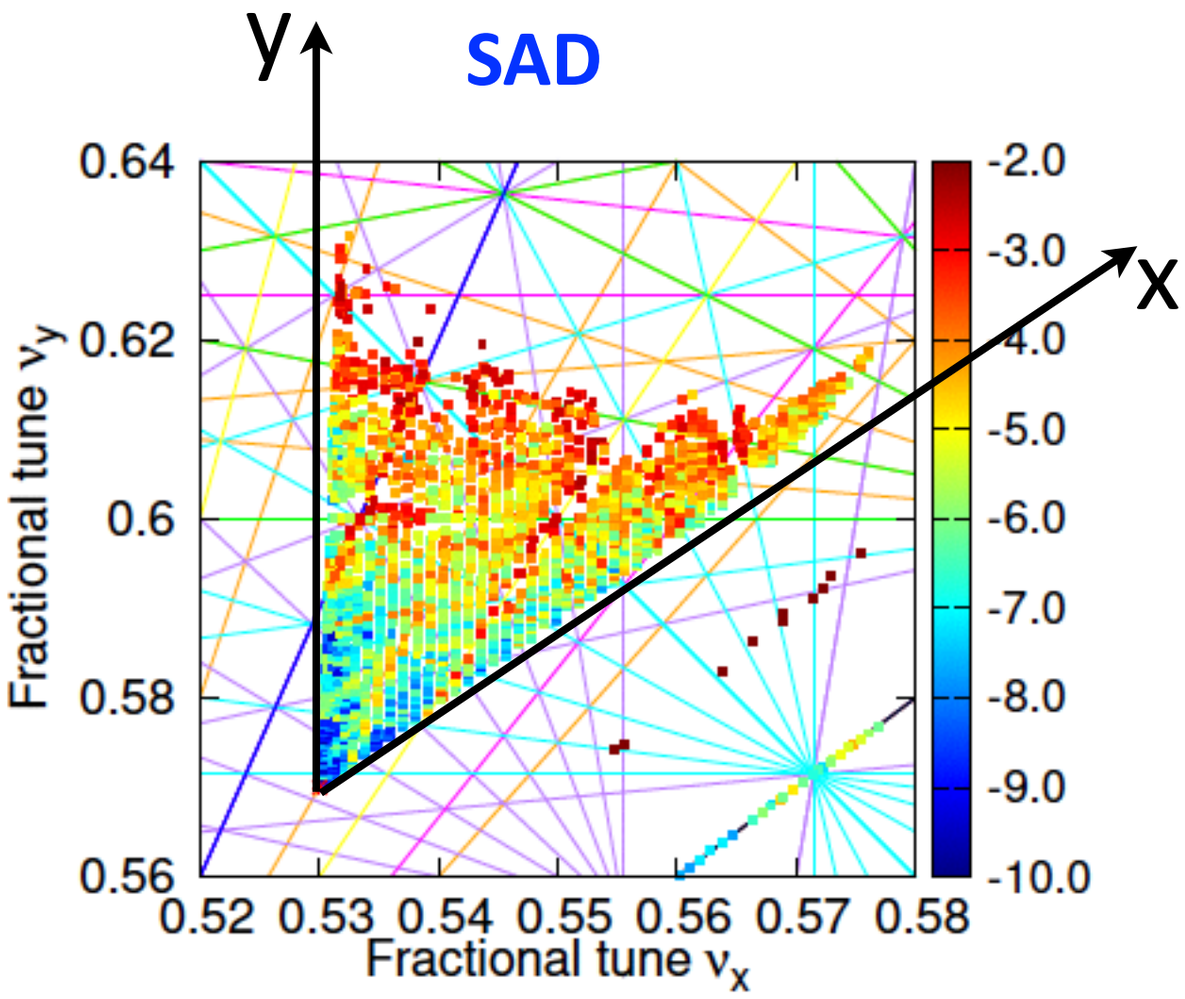


# 2. Comparison of FMA: sler_1684

## ➤ Tune space

- **How to evaluate the discrepancy? [David's answer: Check element-by-element tracking data with large amplitude]**



# 2. Comparison of FMA: sher_5755

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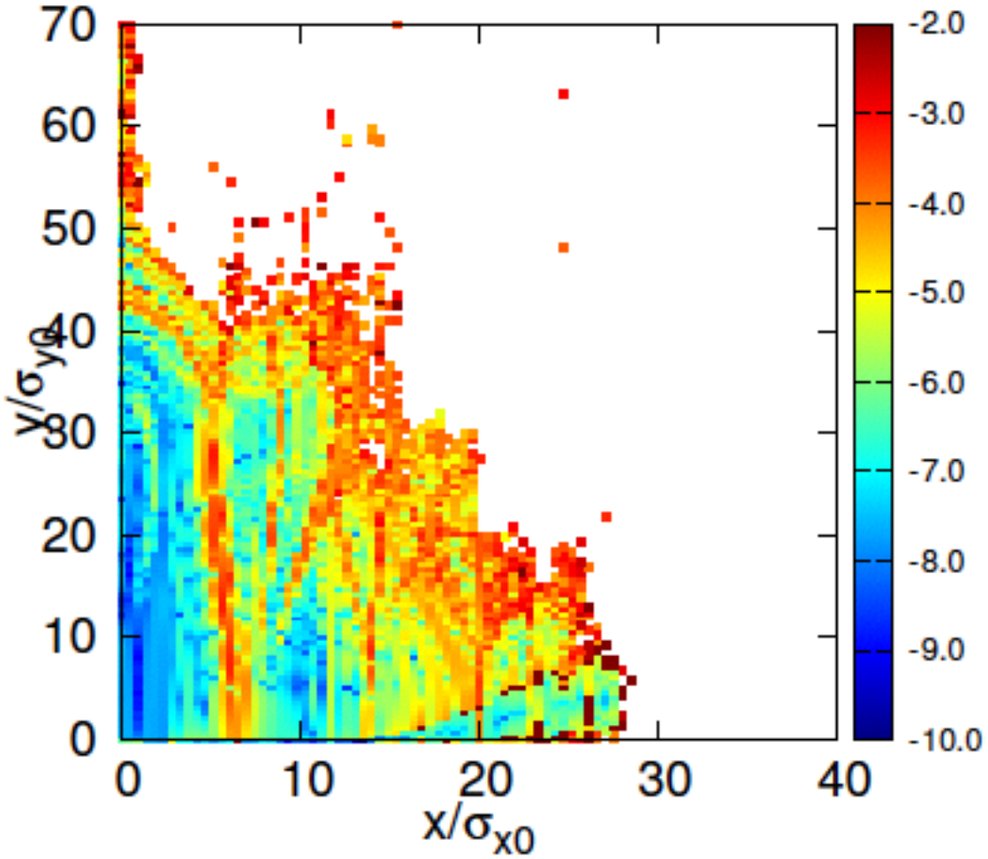
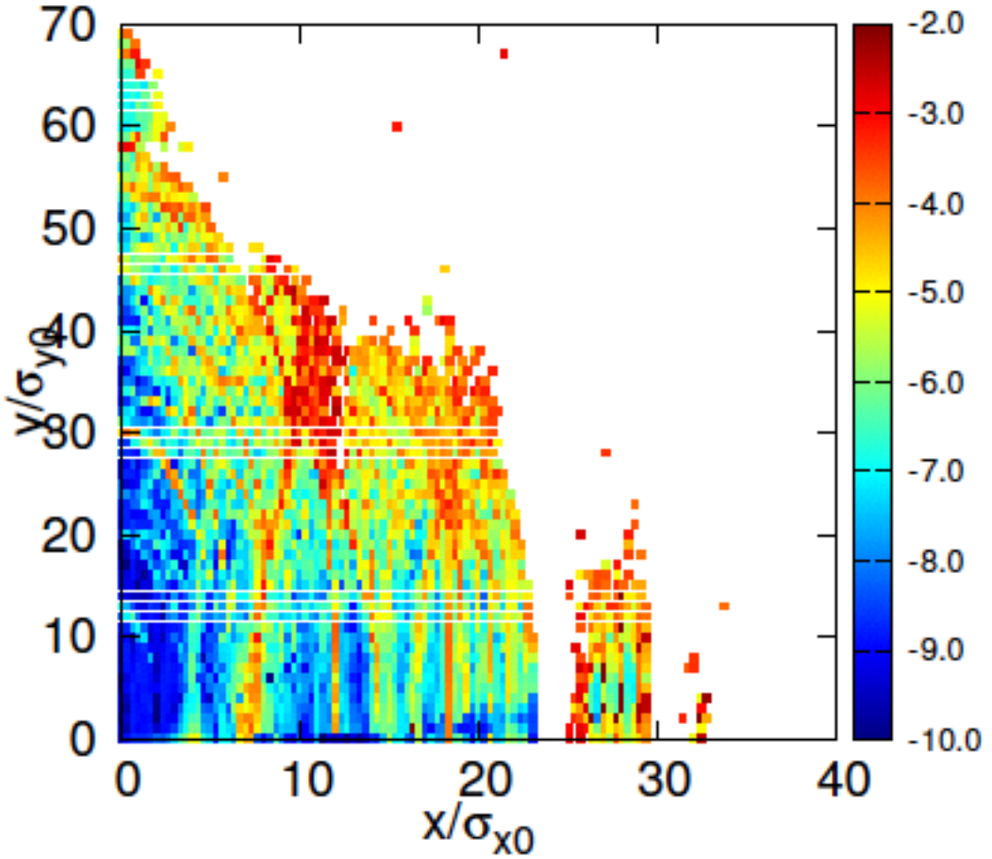
- Discrepancy exists in details: indicate different (high-order) nonlinear maps used by Bmad and SAD?

Bmad




SAD

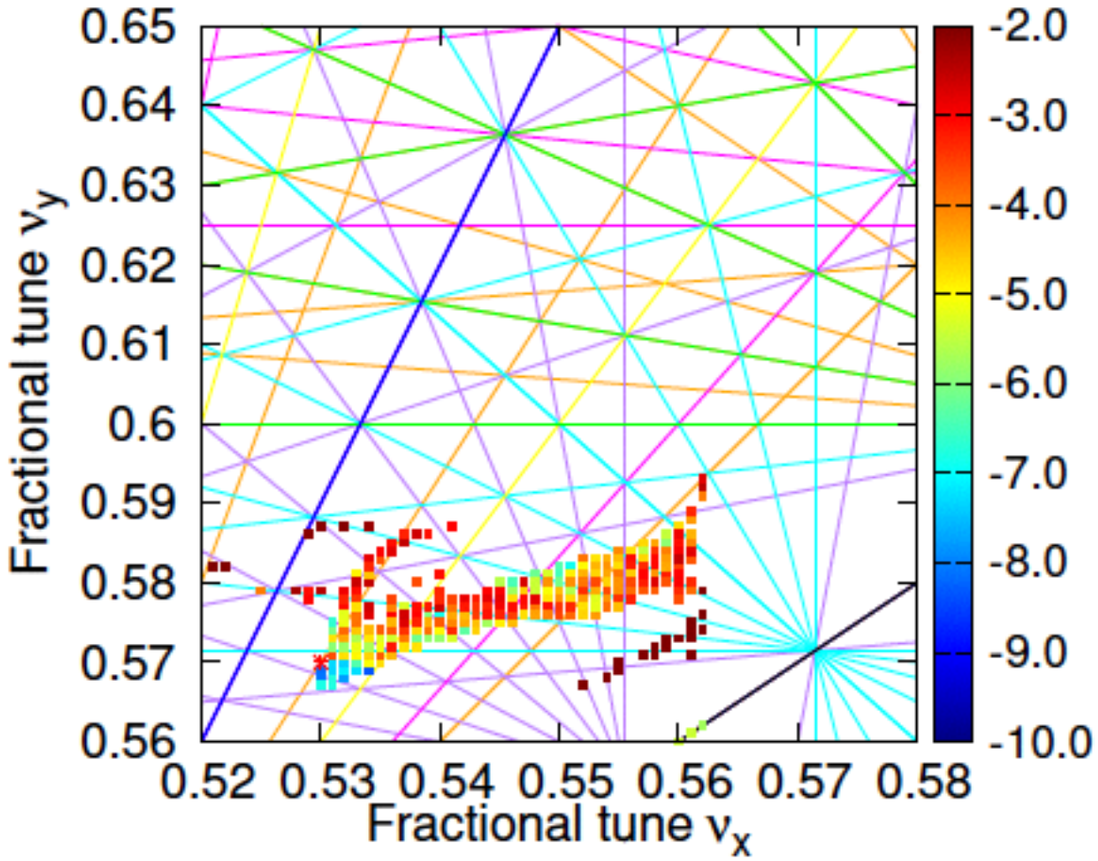


# 2. Comparison of FMA: sher_5755

## ➤ Tune space

- **How to evaluate the discrepancy? [David's answer: Check element-by-element tracking data with large amplitude]**
- **Bmad result looks strange …**

**Bmad**




**SAD**